METRO VANCOUVER PHYSICS CIRCLE 2014-2015
PROBLEM SET 1

Problem 1. A rocket ship is flung (not of its own propulsion) at another planet at a speed of 1000 meters per second and decelerates to 0 by firing its thrusters 100 meters before colliding with the planet. Orders come down from your weird alien overlords that they'd rather the next (identical) rocket decelerate from 200 meters away instead. However, the rocket can only fire its thrusters at one setting. How much faster should the rocket be flung in order to it to come to a clean stop on the planet's surface. Assume that the planet's gravity and the weight of the rocket fuel used is negligible

Solution 1. Let the original speed of the spaceship be $v_0 = 1000\text{m/s}$. Let the original braking distance by $d_0 = 100\text{m}$ and the acceleration of the ship be $-a$.

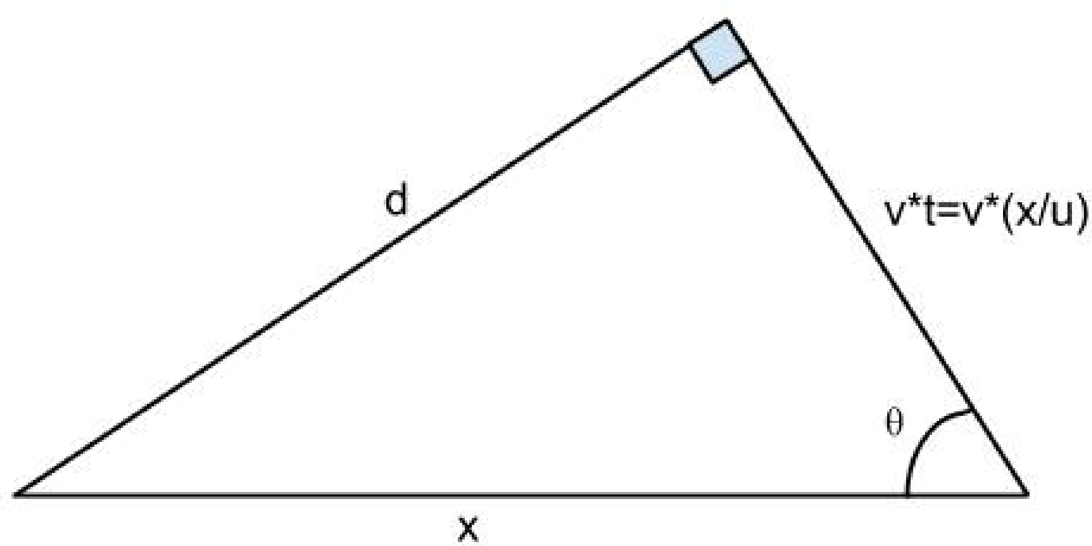
Then, it takes $\frac{v_0}{a}$ time to decellerate. In that time, it will travel a distance of

$$d_0 = v_0 t - \frac{1}{2}at^2 = \frac{v_0^2}{a} - \frac{v_0^2}{2a} = \frac{v_0^2}{2a}$$

Thus, doubling the distance means increasing the velocity by a factor of $\sqrt{2}$ and the rocket should be flung at a speed of $1000\sqrt{2}\text{m/s}$.

Problem 2. Suppose that you are running home in the middle of a rainstorm. Your house is x meters away, and the rain is falling towards your back at a speed of v, $\theta > 0$ degrees (or radians) below the horizontal. How fast should you run (assuming you can run as fast as you'd like), in order to minimize the amount of rain that hits you?

Solution 2. Change your reference frame to the rain's perspective. Then your home is moving upwards and towards you, at a speed of v, θ degrees (or radians) above the horizontal, and you are moving in the at the same rate, plus your running speed horizontally. Now we'd simply like to minimize the distance you travel, as the rain is stationary, thus the further you have to travel, the more rain you move through (hits you). Thus, you'd like to hit your home's path at a right angle. Let u be the speed at which you run. We then end up with the following triangle:



We then find that the length of the path is:

$$d = \sqrt{x^2 - v^2\frac{x^2}{u^2}} = x\sqrt{1 - \frac{v^2}{u^2}} \tag{1}$$

Then, knowing that this is a right triangle, we can use

$$\sin(\theta) = \frac{d}{x} \tag{2}$$

$$= \frac{x\sqrt{1-\frac{v^2}{u^2}}}{x} \tag{3}$$

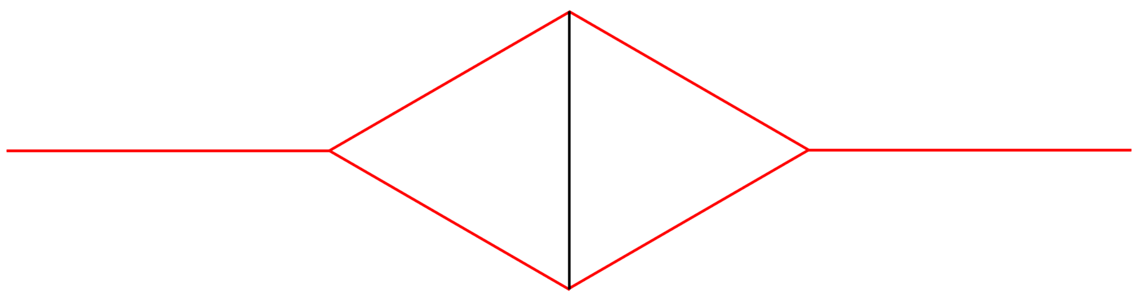
$$= \sqrt{1-\frac{v^2}{u^2}} \tag{4}$$

$$\sin^2(\theta) = 1-\frac{v^2}{u^2} \tag{5}$$

$$1-\sin^2(\theta) = \frac{v^2}{u^2} \tag{6}$$

$$\frac{v}{\sqrt{1-\sin^2(\theta)}} = u \tag{7}$$

Problem 3. In the following diagram, the red lines are springs of spring constant k, while the black line is a rigid bar.



a) Assuming that the length of the bar is negligable compared to the length of the springs, what is the effective spring constant of the result?

b) What happens when you drop the assumption that the length of the bar is negligible?

Solution 3. a) First, we look at what happens when two springs are placed in parallel (two springs beside each other, with their backs and their fronts connected together. Pulling both a distance x away from their equilibrium results in a force of k_1x from the first spring and a force of k_2x from the second. All together, the springs exert $(k_1+k_2)x$ force, for an effective spring constant of k_1+k_2.

Next, we look at what happens when two springs are placed in series (two springs in a row, with the back of one connected to the front of the previous. When the springs are pulled taut, the tension forces at the point in between must cancel. Thus, $k_1x_1 = k_2x_2$. The force at the end of the spring is generated from the tension force, making it also

$$\begin{aligned}
k_2x_2 &= k_{\text{eff}}(x_1+x_2) \\
&= k_{\text{eff}}(\frac{k_2}{k_1}x_2+x_2) \\
\frac{1}{k_{\text{eff}}} &= \frac{1}{k_1}+\frac{1}{k_2} \\
k_{\text{eff}} &= \left(\frac{1}{k_1}+\frac{1}{k_2}\right)^{-1}
\end{aligned}$$

Now, one of the double spring pairs in the center is just springs in parallel and has spring constant $2k$. Either half is just one of these in series with a spring of constant k, for an overall constant of $\left(\frac{1}{k}+\frac{1}{2k}\right)^{-1} = \frac{2k}{3}$. The whole thing is two of these in parallel and has a spring constant of

$$\left(\frac{3}{2k}+\frac{3}{2k}\right)^{-1} = \frac{k}{3}$$

b) Here, Hooke's Law breaks down, but we can still calculate what happens.

First, we look at one of the triangles consisting of two springs and the central bar. Call the equilibrium length of one of the springs x_0 and the length of the seperator bar $2d$ (We assume that $X_0 > d$, or in other words, that the spring is capable of being in equilibrium). Then, the equilibrium distance of the triangle is $\sqrt{x_0^2 - d^2}$, by the Pythagorian Theorem. Let the vertex where the springs meet be pulled outwards a distance of y. Then, each spring is stretched to length

$$\sqrt{\left(\sqrt{x_0^2 - d^2} + y\right)^2 + d^2}$$

. This applies

$$k\left[\sqrt{\left(\sqrt{x_0^2 - d^2} + y\right)^2 + d^2 - x_0}\right]$$

force back along the spring. Then,

$$\frac{\sqrt{\left(\sqrt{x_0^2 - d^2} + y\right)^2}}{\sqrt{\left(\sqrt{x_0^2 - d^2} + y\right)^2 + d^2}} k\left[\sqrt{\left(\sqrt{x_0^2 - d^2} + y\right)^2 + d^2 - x_0}\right]$$

of that force is applied horizontally from each spring, or twice that from both springs put together.

As before, this is the same amount as the tension force in the spring they are connected to, which is displaced a distance z. Thus,

$$kz = 2k\frac{\sqrt{\left(\sqrt{x_0^2 - d^2} + y\right)^2}}{\sqrt{\left(\sqrt{x_0^2 - d^2} + y\right)^2 + d^2}}\left[\sqrt{\left(\sqrt{x_0^2 - d^2} + y\right)^2 + d^2 - x_0}\right] = F$$

where F is the external force applied to the system.

These equations can be solved in for a relationship between $2(y+z)$, the total displacement of the spring system and F, the applied force. However, it will not be a linear relationship. As such Hooke's Law does not apply. When d is taken to be negligably small, however, this should give the same result as in (a).

Problem 4. A yo-yo consists of two solid disks, each of mass150 g and radius5 cm, connected by a central spindle of radius2 cm and negligible mass. A string is coiled around the central spindle. The yo-yo then is placed upright on a rough flat surface and the string is pulled gently with a tension of10 N at an angle30 degrees to the horizontal. The pull is gentle enough to ensure that that yo-yo does not slip nor lifts off the ground.

Find the acceleration of the center of mass of the yo-yo.

Solution 4. We have a balance of torque and force that causes it to accelerate at the same speed it rolls ($a = \alpha R$, where a is the acceleration, α the angular acceleration and R is the radius of the disks). The torque on the yo-yo is given by $F_a r - F_f R$, where $F_a = 10N$ is the force described in the question, F_f is the resulting force of friction, and r and R are the radii of the inner spindle and disks respectively. Then, $\alpha = \frac{F_a r - F_f R}{I}$. Similarly, $a = \frac{\cos\theta F_a - F_f}{m}$, where $\theta = \frac{\pi}{6}$ is the angle with the horizontal. Thus, we get that

$$
\begin{aligned}
a &= \alpha R \\
\frac{\cos\theta F_a - F_f}{m} &= \frac{F_a r - F_f R}{I} R \\
F_f \left(\frac{1}{m} - \frac{R^2}{I} \right) &= F_a \left(\frac{rR}{I} - \frac{1}{m} \right) \\
F_f \left(I - R^2 m \right) &= F_a \left(rRm - I \right) \qquad \text{Note: } I = \frac{1}{2} m R^2 \text{ for discs} \\
-\frac{1}{2} F_f &= F_a \left(\frac{r}{R} - \frac{1}{2} \right) \\
&= -\frac{1}{10} F_a \\
F_f &= \frac{1}{5} F_a \\
a &= \frac{\cos\theta F_a - F_f}{m} \\
&= \frac{\left(\frac{\sqrt{3}}{2} - \frac{1}{5} \right) 10N}{.15kg} \\
&\approx 44 m/s^2
\end{aligned}
$$